## Personal Perspective of Aeroelasticity During the Years 1953–1993

John Dugundji

Massachusetts Institute of Technology, Cambridge, Massachusetts 02139-4307

A brief review is given of the author's personal impressions of the development of aeroelasticity during the sixth to ninth decades of flight. It is not intended to be all-inclusive, but rather a brief look at the field as the author saw it, from his own personal viewpoint as a worker in this area. It is limited to aircraft problems and does not embrace the parallel and large field of aeroelastic problems in helicopters.

The field of aeroelasticity encompasses the interaction of aerodynamic, elastic, and inertia forces acting on a flight vehicle. This was evident in the early days of flight, even at low speeds, because aircraft structures were relatively flexible. As aircraft speeds increased and knowledge of aeroelasticity became more widespread, aircraft structures were made stiffer and were better designed to avoid these aeroelastic interaction problems. During World War II, with the advent of high speeds and slender-wing aircraft, a number of significant aeroelastic problems began to appear, ranging from loss of aileron power and control reversal, to wing and control surface flutter. The postwar period, with the emergence of highspeed jet transports with swept wings and engine nacelles, led to careful studies of the effects of aeroelasticity in the initial design of these aircraft. The basic approach to aeroelastic effects became centered around four general areas: 1) the development of effective analytic and numerical studies for flutter and aeroelasticity, 2) wind-tunnel testing of dynamically scaled wind tunnel models, 3) ground vibration tests to check the natural frequencies and stiffness properties of the actual aircraft, and 4) careful techniques for flight testing the actual aircraft. By the mid-1950s, aeroelasticity had become recognized as an important part of the aircraft design process. An excellent historical review of developments in aeroelasticity up to this period, may be found by Garrick and Reed.<sup>1</sup> A seminal paper that marked the emergence of the field was given by Collar<sup>2</sup> in 1946. Another notable review emphasizing the British efforts in aeroelasticity up to this period was also given by Collar.3

By the time of the sixth decade of flight, it seemed aeroelasticity had reasonably matured. Much wind-tunnel flutter model testing and research had been conducted at NACA and other laboratories. Analyses had been codified, certification procedures had been established, and much experience had been gained in theoretical analysis, wind-tunnel testing, ground vibrations surveys, and cautious

flight testing. The analytical methods were usually centered around two-, three-, or four-degree-of-freedomflutter analyses focused on separate wing or tail components. These were generally organized into efficient matrix formulations, and the computations were often done by a dedicated computation group on mechanical desk computers. With good insight into the significant vibration modes of an aircraft, knowledge based on past experience, and wind-tunnel flutter model testing of specific and general aircraft components, one could obtain a good approximation to the aeroelastic behavior of an aircraft. Several notable textbooks on aeroelasticity and flutter were published about this time, among which were Scanlan and Rosenbaum, Fung, and Bisplinghoff et al. The latter textbook, particularly, summarized the state of the art at this time and remained a classic reference for many years.

Yet, there were considerable changes to come in the field. These were basically associated with the introduction of the digital computer in the mid-1950s, and the rapid increase in computational power that this brought about. This has affected every aspect of aeroelasticity down to the present day. One of the first notable changes brought about by computers was the emergence of the finite element method for structural computation.<sup>7–9</sup> Finite element stiffness methods had been formulated earlier as a simple routine way of describing structures, but only became practical with the availability of large computers. Similarly, panel surface area methods for describing air forces became practical, 10,11 thereby replacing the cruder two-dimensional strip theory methods that had been used earlier. Computational fluid dynamics (CFD) emerged as a practical technique for numerically solving the partial differential equations of fluid flow in the compressible flow regimes. 12,13 Larger aeroelastic systems with many more degrees of freedom could be analyzed at the same time, rather than looking at selected components.<sup>14</sup> Entire flutter programs could be formulated and mechanized for optimization and tradeoff studies.<sup>15,16</sup> Control systems characteristics were formulated and incorporated conveniently into the aircraft's flutter and random gust response behavior.<sup>17,18</sup> Additionally, the emergence of the digital computer and associated analog-to-digital converters affected tremendously obtaining, storing, and processing of experimental data for aeroelastic tests. <sup>19,20</sup> The field of aeroelasticity was greatly transformed by the advent and growth of the digital computer.



John Dugundjiis a Professor of Aeronautics and Astronautics Emeritus at Massachusetts Institute of Technology. He received his B.A.E. degree from New York University and S.M. and Sc.D. degrees in aeronautical engineering from the Massachusetts Institute of Technology. His industrial experience includes one and one-half years with Grumman Aircraft Engineering Corporation on aerodynamic research problems and four years at Republic Aviation Corporation on theoretical and experimental methods of flutter, vibration, and aeroelastic dynamic load problems. He has been on the faculty of the Department of Aeronautics and Astronautics from 1957 to 1993. His general interests are in aeroelasticity, structural dynamics, applied mechanics, and composite material structures. He is an Associate Fellow of AIAA.

Received 13 November 2002; revision received 25 February 2003; accepted for publication 13 May 2003. Copyright © 2003 by John Dugundji. Published by the American Institute of Aeronautics and Astronautics, Inc., with permission. Copies of this paper may be made for personal or internal use, on condition that the copier pay the \$10.00 per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923; include the code 0021-8669/03 \$10.00 in correspondence with the CCC.

810 DUGUNDJI

A brief personal recollection by the author of some specific topics in aeroelasticity that emerged during the sixth to ninth decades of flight are described next. These are listed roughly in chronological order as recalled by the author.

In the middle 1950s, the effects of aerodynamic heating on structures and aeroelasticity were brought up in a paper by Bisplinghoff on high-speedflight.<sup>21</sup> These were further explored, but it was found that they could be treated by considering the loss of stiffness in the structure due to thermal stresses and reduced material properties. Careful consideration of transient and steady-state temperature distributions were involved, which could be obtained for arbitrary structural configurations by finite element modeling for heating.<sup>22</sup>

Additional interesting insights into the mechanism of flutter appeared in the late 1950s. These centered around investigations of the simple typical section bending—torsion flutter case and showed that flutter could occur even in the absence of damping forces because of the coalescence of the bending and torsion natural frequencies. <sup>23–25</sup>

The flutter of skin panels at supersonic speeds became an interesting topic of study in the early 1960s. 26,27 It had a simple, easily defined instability mechanism, and gave rise to traveling wave motions. Although not a critical flight problem, it allowed interesting investigations into structural nonlinearity, limit cycles, chaotic motion, and anisotropic plate effects. 28,29

Propeller whirl flutter was encountered in some actual flight incidents with a turboprop aircraft and actually led to two crashes. Subsequent analytic and experimental wind-tunnel model investigations pointed to weakened engine mountings as the cause because they allowed the coupling of the gyroscopic propeller forces with engine nacelle and wing motions into a dangerous flutter situation.<sup>30</sup>

As mentionedearlier, the growth of computational power allowed much better unsteady pressure distributions to be represented over an oscillating wing surface, both chordwise and spanwise. An early attempt for obtaining pressure distributions in supersonic flow was the box method. A later method that evolved in the late 1960s was the doublet-lattice method, which could be used for harmonically oscillating motions into the high subsonic range. This proved very popular because of its engineering simplicity. Other methods based on using Green's function also found favor (see Ref. 11) Later, with expanded computing power, CFD solutions of the Euler equations of fluid motion became possible, and even CFD solutions of the viscous Navier–Stokes equations were attempted. Also, numerical solutions of the potential flow equation over the entire aircraft surface were developed, to attempt approximate solutions into the nonlinear transonic range.

By the early 1970s, the aeroelastic representation of a complete aircraft could be represented by a finite element structural model together with a set of aerodynamic surface panels over the entire aircraft. Such a discrete model could be conveniently analyzed for static aeroelastic response and behavior.<sup>34</sup> For dynamic response and flutter, however, the model would have to be reduced to a much smaller number of significant vibration modes of interest.<sup>35,36</sup> The art and experience of the engineer would have to be involved here to choose the appropriate modes for the analysis.

By the middle 1970s, the analysis of control systems had reached a refined state. Control systems were generally cast into state-space form involving systems of first-order differential equations rather than second-order mass-spring-damping systems. To mate with control systems in the time domain, aircraft flutter analyses were often reformulated from the frequency domain to the more versatile time domain.<sup>37</sup> This involved fitting the easier-to-obtain harmonically oscillating airforces with Padé approximations for the time domain and led to convenient transient response and root locus stability interpretations of flutter and control system interactions.<sup>38,39</sup> Also, the powerful root finding methods of state space with large computers could be invoked. For response to random gusts, the effects of linear Gaussian quadratic and other types of controllers could be integrated into the aircraft response.<sup>40</sup>

With the increased use of turbofan engines, the flutter of large compressor fan blades became of significant interest. Because of the many closely spaced blades and the high mass density ratios of the blades, these were often analyzed as single-degree-of-frædom

systems, in which the interblade phase angle between the motion of the blades played a key role in the flutter behavior. <sup>41</sup> New analyses in the early 1980s allowed for independent bending and torsional motions of the blades and looked at the coupling of different circumferential traveling wave modes in a rotating frame. <sup>42</sup> An alternate viewpoint using conventional standing wave modes in a fixed frame, vibrating 90 deg out of phase with each other, produced similar results. <sup>43</sup> Different aerodynamic theories were used to help explore the various regimes of flutter in axial compressors. <sup>41</sup> Also the effects of mistuned blades on both the flutter and vibrational characteristics was investigated. <sup>44</sup>

In the late 1970s, composite structures made of graphite–epoxy materials began to be considered because of their light weight and high stiffness and strength characteristics. By arranging ply layups, one could also obtain bending–torsion and extension–torsion couplings. It was proposed that for sweptforward wings, favorable bending–torsion structural coupling in the wings could offset the statically unstable aeroelastic behavior of sweptforward wings. This prompted much exploration of the static divergence and flutter behavior of these composite tailored wings. Also, the experimental X-29 was built. An interesting consequence of aircraft with sweptforward wings was the possible onset of body freedom flutter, involving coupling of the lowered wing bending modes with the rigid-body pitch and translation modes of the aircraft. Later applications of composite materials focused on designing the wings to achieve certain optimum shapes during the flight regimes.

The use of active structural elements such as piezoelectric materials began to appear in the middle 1980s.<sup>48</sup> The concept of selectively warping a wing to minimize its gust response and extend its flutter boundary was shown in several wind-tunnel tests on small-and medium-size models.<sup>49</sup> Also, the concept was examined for controlling the torsional behavior of helicopter blades.<sup>50</sup> Another application of active elements was to minimize the vibration and noise of a panel by selectively activating groups of piezoelectric elements over its surface.

Nonlinearities are inherent in much aeroelastic behavior, particularly when steady oscillations are observed. Linear theory is usefulin predicting the stability boundary and the growth of oscillations, but does not predict the final amplitude in cases where the flutter is mild. Also, nonlinearities can play a dangerous role in that disturbances of sufficient magnitude may excite a flutter condition, whereas smaller disturbances do not. Some early examples of nonlinear effects on flutter were examined in the frequency domain using describing functions to model the effects of free play or Coulomb friction in a control system.<sup>51,52</sup> Later investigations included the effects of large static wing deflections,<sup>53–55</sup> aerodynamic stall effects on flutter,<sup>56,57</sup> and the earlier mentioned transonic flow regime.<sup>33</sup> With the increase of computing power, many of these nonlinear effects could now be examined in the time domain, rather than in the frequency domain as formerly. The use of the time domain also can include the nonlinear effects of chaos, which cannot be found from investigations in the frequency domain.58

Much progress has been made over the years in obtaining and processing experimental test data. In the middle 1950s, most dynamic data were taken with oscillographs, which had to be subsequently read, and whose timing speeds had to be adjusted carefully to record the frequencies of interest. With the advent of large digital computers and analog-to-digital converters, one could store and process very large amounts of test data and redisplay that data for the desired timescales. Furthermore, one could statistically and Fourier analyze the data in real time, while the experiment was proceeding. This made huge improvements possible in wind-tunnel testing, ground vibration testing, and flight testing, particularly because digital computers became more powerful, faster, and smaller in size every year. However, as in all experimental testing, experience and art still played a large role in obtaining and interpreting the significant test data present amidst the generally accompanying noise. In connection with wind-tunnel testing, a major experimental facility was built at NASA Langley Research Center in 1960, the Transonic Dynamics Tunnel (TDT), for the exclusive use of aeroelasticity research. This  $16 \times 16$  ft transonic tunnel proved very valuable DUGUNDJI 811

Table 1 Approximate time line of research areas in aeroelasticity

Year	Research area
1950	General introduction of digital computers
	Effects of aerodynamic heating
	Additional insights into mechanisms of flutter
	Flutter of skin panels
1960	Finite element methods for structures
	Propeller whirl flutter
	Surface panel methods for air forces
1970	Complete aircraft representation by structural
	elements and aerodynamic panels
	CFD.
	State-space representation of control systems
	Structural nonlinearities (stiffening springs, dead zones, etc.)
1980	Digital data gathering techniques
	Introduction of composites and aeroelastic tailoring
	Flutter of compressor fan blades
	Piezoelectric actuators and active control
1990	Nonlinearities and limit cycles (large geometric deflections,
	stall flutter, transonic flow, etc.)

and supported much research and development of specific aircraft over the years.<sup>19</sup> An interesting 14-min film showing some aeroelastic testing at the TDT and other flutter occurrences was made by NASA in 1973.<sup>59</sup>

This paper is a brief recollection of the author's personal impressions of aeroelasticity as he saw the field develop during the sixth to ninth decades of flight. It is limited to aircraft problems. An approximate time line summary of these research areas in aeroelasticity is presented in Table 1. Some of these later developments were foreseen in an interesting paper by Garrick in 1976. A standard textbook describing various aspects of aeroelasticity during these years was written by Dowell et al. Some current problems and trends in aeroelsticity at the start of the second century of flight can be found in published articles of this and other aerospace journals.

## References

<sup>1</sup>Garrick, I. E., and Reed, W. H., "Historical Development of Aircraft Flutter," *Journal of Aircraft*, Vol. 18, No. 11, 1981, pp. 897–912.

<sup>2</sup>Collar, A. R., "The Expanding Domain of Aeroelasticity," *Journal of the Royal Aeronautical Society*, Vol. 51, Jan. 1947, pp. 1–34.

<sup>3</sup>Collar, A. R., "The First Fifty Years of Aeroelasticity," *Aerospace*, Vol. 5, Feb. 1978, pp. 12–20.

<sup>4</sup>Scanlan, R. H., and Rosenbaum, R., Introduction to the Study of Aircraft Vibration and Flutter, MacMillan, London, 1951.

<sup>5</sup>Fung, Y. C., An Introduction to the Theory of Aeroelasticity, Wiley, New York, 1955.

<sup>6</sup>Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., *Aeroelasticity*, Addison–Wesley, Reading, MA, 1955.

<sup>7</sup>Argyris, J. H., "The Impact of the Digital Computer on Engineering Sciences," Twelfth Lancaster Memorial Lecture, *Aeronautical Journal of the Royal Aeronautical Society*, Vol. 74, Jan.–Feb. 1970, pp. 13–41, 111–127.

<sup>8</sup>Turner, M. J., Clough, R. W., Martin, H. C., and Topp, L. J., "Stiffness and Deflection Analysis of Complex Structures," *Journal of the Aeronautical Sciences*, Vol. 23, No. 9, 1956, p. 805.

<sup>9</sup>Przemieniecki, J. S., *Theory of Matrix Structural Analysis*, McGraw–Hill, New York, 1968.

<sup>10</sup> Albano, E., and Rodden, W. P., "A Doublet-Lattice Method for Calculating Lift distribution on Oscillating Surfaces in Subsonic Flows," *AIAA Journal*, Vol. 7, No. 2, 1969, pp. 279–285.

<sup>11</sup>Morino, L., Chen, L., and Susiu, E. O., "Steady and Oscillatory Subsonic and Supersonic Aerodynamics Around Complex Configurations," *AIAA Journal*, Vol. 13, No. 3, 1975, pp. 368–374.

<sup>12</sup>Borland, C. J., and Rizzetta, D. P., "Nonlinear Transonic Flutter Analysis," AIAA Journal, Vol. 20, No. 11, 1982, pp. 1606–1615.

ysis," AIAA Journal, Vol. 20, No. 11, 1902, pp. 1000 1011.

13 Guruswamy, G. P., "Unsteady Aerodynamic and Aeroelastic Calculations for Wings Using Euler Equations," AIAA Journal, Vol. 28, No. 3, 1990, pp. 461–469.

14 Kussner, H. G., "A Comparison of Methods Used in Flutter Research,"

<sup>14</sup>Kussner, H. G., "A Comparison of Methods Used in Flutter Research," AGARD Rept. 592, Supplement, *AGARD Manual of Aeroelasticity*, Vol. 6, Aug. 1972.

Aug. 1972.

15 Schmit, L. A., "Structural Synthesis—Its Genesis and Development,"

AIAA Journal, Vol. 19, No. 11, 1981, pp. 1249–1263.

<sup>16</sup>Wilkinson, K., Markowitz, J., Lerner, E., and George, D., "Application of the Flutter and Strength Optimization Program (FASTOP) to the Sizing of Metallic and Composite Lifting-Surface Structures," *Proceedings AIAA/ASME/SAE 17th Structures, Structural Dynamics, and Materials Conference*, AIAA, New York, May 1976, pp. 463–472.

<sup>17</sup>Newsome, J. R., "Control Law Synthesis for Active Flutter Suppression Using Optimum Control," *Journal of Guidance and Control*, Vol. 2, No. 5, 1979, pp. 388–394.

<sup>18</sup>Karpel, M., "Design for Active Flutter Suppression and Gust Alleviation Using State-Space Aeroelastic Modeling," *Journal of Aircraft*, Vol. 19, No. 3, 1982, pp. 221–227.

<sup>19</sup>Ricketts, R. H., "Experimental Aeroelasticity, History, Status and Future in Brief," NASA-TM-102651, April 1990; also AIAA Paper 90-0978, April 1990.

<sup>20</sup>Roger, K. L., Hodges, G. E., and Felt, L., "Active Flutter Suppression: A Flight Test Demonstration," *Journal of Aircraft*, Vol. 12, No. 6, 1975, pp. 551–556.

pp. 551–556.

<sup>21</sup>Bisplinghoff, R. L., "Some Structural and Aeroelastic Considerations of High Speed Flight," Nineteenth Wright Brothers Lecture, *Journal of the Aeronautical Sciences*, Vol. 23, No. 4, 1956, pp. 289–329.

<sup>22</sup>Thornton, E., "Thermal Structures: Four Decades of Progress," *Journal of Aircraft*, Vol. 29, No. 3, 1992, pp. 485–497.

<sup>23</sup>Pines, S., "An Elementary Explanation of the Flutter Mechanism," *Proceedings of National Specialists Meeting on Dynamics and Aeroelasticity*, Inst. of the Aeronautical Sciences, New York, 1958, pp. 52–58.

<sup>24</sup>Bisplinghoff, R. L., and Ashley, H., *Principles of Aeroelasticity*, Wiley, New York, 1962, Chap. 6.

<sup>25</sup>Dugundji, J., "Effect of Quasi-Steady Air Forces on Incompressible Bending-Torsion Flutter," *Journal of the Aeronautical Sciences*, Vol. 25, No. 2, 1958, pp. 119–121.

<sup>26</sup>Dowell, E. H., *Aeroelasticity of Plates and Shells*, Noordhoff, Leyden, The Netherlands, 1975.

<sup>27</sup>Dugundji, J., "Theoretical Considerations of Panel Flutter at High Supersonic Mach Numbers," *AIAA Journal*, Vol. 4, No. 7, 1966, pp. 1257–1266.

<sup>28</sup>Dowell, E. H., "Nonlinear Oscillations of a Fluttering Plate," *AIAA Journal*, Vol. 4, No. 7, 1966, pp. 1267–1275.

<sup>29</sup>Sawyer, J. W., "Flutter and Buckling of General Laminated Plates," *Journal of Aircraft*, Vol. 14, No. 4, 1977, pp. 387–393.

<sup>30</sup>Reed, W. H., III, "Review of Propeller-Rotor Whirl Flutter," NASA TR R-264, July 1967.

<sup>31</sup>Pines, S., Dugundji, J., and Neuringer, J., "Aerodynamic Flutter Derivatives for a Flexible Wing with Supersonic and Subsonic Edges," *Journal of the Aeronautical Sciences*, Vol. 22, No.10, 1955, pp. 693–700.

<sup>32</sup>Lee-Rausch, E. M., and Batina, J. T., "Wing Flutter Computations Using an Aerodynamic Model Based on the Navier–Stokes Equations," *Journal of Aircraft*, Vol.33, No. 6, 1996, pp. 1139–1147.

<sup>33</sup>Batina, J. T., Seidel, D. A., Bland, S. R., and Bennett, R. M., "Unsteady Transonic Flow Calculations for Realistic Aircraft Configurations," *Journal of Aircraft*, Vol. 26, No. 1, 1989, pp. 21–28.

<sup>34</sup>Stauffer, W. A., Lewolt, J. G., and Hoblit, F. M., "Application of Advanced Methods to Design Loads Determination for the L-1011 Transport," *Journal of Aircraft*, Vol. 10, No. 8, 1973, pp. 149–158.

<sup>35</sup>Guyan, R. J., "Reduction of Stiffness and Mass Matrices," *AIAA Journal*, Vol. 3, No. 2, 1965, p. 380.

<sup>36</sup>Craig, R. R., Jr., and Bampton, M. C. C., "Coupling of Substructures for Dynamic Analysis," *AIAA Journal*, Vol. 6, No. 7, 1968, pp. 1313–1319.

<sup>37</sup>Edwards, J. W., Ashley, H., and Breakwell, J. V., "Unsteady Aerodynamic Modeling for Arbitrary Motions," *AIAA Journal*, Vol. 17, No. 4, 1979, pp. 365–374.

<sup>38</sup>Vepa, R., "Finite State Modeling of Aeroelastic Systems," NASA CR-2779, 1977; also, AIAA Paper 76-17, 1976.

<sup>39</sup>Abel, I., "An Analytic Technique for Predicting the Characteristics of a Flexible Wing Equipped with an Active Flutter-Suppression System and Comparison with Wind-Tunnel Data," NASA TP 1367, Feb. 1979.

<sup>40</sup>Noll, T. E., Eastep, F. E., and Calico, R. A., "Active Suppression of Aeroelastic Instabilities on a Forward Swept Wing," *Journal of Aircraft*, Vol. 21, No. 3, 1984, pp. 202–208.

<sup>41</sup>Mikolajczak, A. A., Arnoldi, R. A., Snyder, L. E., and Stargardter, H., "Advances in Fan and Compressor Blade Flutter Analysis and Predictions," *Journal of Aircraft*, Vol. 12, No. 4, 1975, pp. 325–332.

<sup>42</sup>Kaza, K. R. V., and Kielb, R. E., "Flutter and Response of a Mistuned Cascade in Incompressible Flow," *AIAA Journal*, Vol. 20, No. 8, 1982, pp. 1120–1127.
 <sup>43</sup>Dugundji, J., and Bundas, D. J., "Flutter and Forced Response of Mis-

<sup>43</sup>Dugundji, J., and Bundas, D. J., "Flutter and Forced Response of Mistuned Rotors Using Standing Wave Analysis," *AIAA Journal*, Vol. 22, No. 11, 1984, pp. 1652–1661.

812 DUGUNDJI

<sup>44</sup>Crawley, E. F., and Hall, K. C., "Optimization and Mechanisms of Mistuning in Cascades," Journal of Engineering for Gas Turbines and Power, Vol. 107, April 1985, pp. 418-426.

<sup>45</sup>Weisshaar, T. A., "Divergence of Forward Swept Composite Wings," Journal of Aircraft, Vol. 17, No. 6, 1980, pp. 442-448.

<sup>46</sup>Shirk, M. H., Hertz, T. J., and Weisshaar, T. A., "Aeroelastic Tailoring: Theory, Practice, and Promise," Journal of Aircraft, Vol. 23, No. 1, 1986,

pp. 6-18. <sup>47</sup>Chen, G. S., and Dugundji, J., "Experimental Aeroelastic Behavior of

Forward-Swept Graphite/Epoxy Wings with Rigid-Body Freedom," Journal

of Aircraft, Vol. 24, No. 7, 1987, pp. 454-462. <sup>48</sup>Crawley, E. F., and de Luis, J., "Use of Piezoelectric Actuators as El-

ements of Intelligent Structures," AIAA Journal, Vol. 25, No. 10, 1987, pp. 1373–1385.

<sup>49</sup>Lin, C. Y., Crawley, E. F., and Heeg, J., "Open- and Closed-Loop Results

of a Strain Actuated Active Aeroelastic Wing," Journal of Aircraft, Vol. 33, No. 5, 1996, pp. 987-994.

<sup>50</sup>Rodgers, J. P., and Hagood, N. W., "Design, Manufacture, and Testing of an Integral Twist-Actuated Rotor Blade," Proceedings of the 8th International Conference on Adaptive Structures and Technologies, Technomic Publishing Co., Lancaster, PA, 1998, pp. 63–72.  $^{51}$  Breitbach, E., "Effects of Structural Nonlinearities on Aircraft Vibration

and Flutter," NATO AGARD Rept. R-665, Jan. 1978.

<sup>52</sup>Sinha, A., and Griffin, J. H., "Friction Damping of Flutter in Gas Turbine Engine Airfoils," Journal of Aircraft, Vol. 20, No. 4, 1983, pp. 372-376.

<sup>53</sup>Minguet, P., and Dugundji, J., "Experiments and Analysis for Composite Blades Under Large Deflections, Part 1: Static Behavior," AIAA Journal, Vol. 28, No. 9, 1990, pp. 1573-1579.

<sup>54</sup>Minguet, P., and Dugundi, J., "Experiments and Analysis for Composite Blades Under Large Deflections, Part 2: Dynamic Behavior," AIAA Journal, Vol. 28, No. 9, 1990, pp. 1580-1589.

<sup>55</sup>Atilgan, A. R., and Hodges, D. H., "Unified Nonlinear Analysis for Nonhomogeneous Anisotropic Beams with Closed Cross Sections," AIAA Journal, Vol. 29, No. 11, 1991, pp. 1990–1999.

56 Dunn, P., and Dugundji, J., "Nonlinear Stall Flutter and Divergence

Analysis of Cantilevered Graphite/Epoxy Wings," AIAA Journal, Vol. 30, No. 1, 1992, pp. 153-162.

<sup>57</sup>Tang, D. M., and Dowell, E. H., "Experimental and Theoretical Study for Nonlinear Aeroelastic Behavior of a Flexible Rotor Blade," AIAA Journal, Vol. 31, No. 6, 1993, pp. 1133-1142.

<sup>58</sup> Virgin, L.N., and Dowell, E. H., "Nonlinear Aeroelasticity and Chaos," Computational Nonlinear Mechanics in Aerospace Engineering, edited by S. N. Atluri, Progress in Aeronautics and Astronautics Series, Vol. 146, AIAA, Washington DC, 1992, Chap. 15.

<sup>59</sup> "Aeroelastic Phenomena and Related Research," NASA Technical Film Serial, L-1128, June 1973.

60 Garrick, I. E., "Aeroelasticity: Frontiers and Beyond," Journal of Air-

*craft*, Vol. 13, No. 9, 1976, pp. 641–657.

<sup>61</sup>Dowell, E. H., Crawley, E. F., Curtiss, H. C., Jr., Peters, D. A., Scanlan, R. H., and Sisto, F., A Modern Course in Aeroelasticity, edited by E. H. Dowell, 3rd ed., Kluwer Academic, Dordrecht, The Netherlands,